

A MATRICES (6)

Information store

Matrices are sometimes used as a short, compact way of storing, or showing, information, e.g. Garside School has four soccer teams: 1st, 2nd, 3rd and Junior. Their results last season (win, lose or draw) can be shown as a matrix

$$\begin{matrix} & \begin{matrix} 1st & 2nd & 3rd & Jun \end{matrix} \\ \begin{matrix} W \\ L \\ D \end{matrix} & \begin{pmatrix} 9 & 3 & 4 & 6 \\ 5 & 4 & 4 & 0 \\ 2 & 3 & 1 & 3 \end{pmatrix} \end{matrix} \quad \begin{matrix} \text{This matrix shows that the 1st} \\ \text{team won 9 times, lost 5 times} \\ \text{and drew twice, etc.} \end{matrix}$$

In the local schools league, a win counts 3 points, a loss counts 1 point and a draw counts 2 points. If the results matrix is premultiplied by $\begin{pmatrix} 3 & 1 & 2 \end{pmatrix}$, a new matrix is obtained.

$$\begin{matrix} & \begin{matrix} W & L & D \end{matrix} & & \begin{matrix} 1st & 2nd & 3rd & Jun \end{matrix} \\ \text{Points} & \begin{pmatrix} 3 & 1 & 2 \end{pmatrix} & \begin{matrix} W \\ L \\ D \end{matrix} & \begin{pmatrix} 9 & 3 & 4 & 6 \\ 5 & 4 & 4 & 0 \\ 2 & 3 & 1 & 3 \end{pmatrix} & = & \text{Points} & \begin{pmatrix} 36 & 19 & 18 & 24 \end{pmatrix} \end{matrix}$$

The new matrix shows the total points gained by each team, e.g. 1st team 36 points, etc.

If, instead, the results matrix were premultiplied by $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ what information would the new matrix give?

e.g. (2) The Walker family buy bottles of milk, tubs of cream and cartons of yogurt from the milkman. The items they bought last week can be shown as a 3×6 matrix

$$\begin{matrix} \text{(M=Monday, etc.)} & & \begin{matrix} M & Tu & W & Th & F & S \end{matrix} \\ \begin{matrix} \text{Milk} \\ \text{Cream} \\ \text{Yogurt} \end{matrix} & \begin{pmatrix} 4 & 3 & 3 & 2 & 4 & 6 \\ 2 & 0 & 2 & 0 & 1 & 2 \\ 8 & 4 & 6 & 4 & 0 & 10 \end{pmatrix} \end{matrix}$$

This matrix shows that on Monday the Walkers bought 4 bottles of milk, 2 tubs of cream and 8 cartons of yogurt, etc.

Milk costs 30p a bottle, cream costs 42p a tub and yogurt costs 25p a carton. Premultiplying by $\begin{pmatrix} 30 & 42 & 25 \end{pmatrix}$ gives the total cost, in pence, for each day's supply

$$\begin{pmatrix} 30 & 42 & 25 \end{pmatrix} \begin{pmatrix} 4 & 3 & 3 & 2 & 4 & 6 \\ 2 & 0 & 2 & 0 & 1 & 2 \\ 8 & 4 & 6 & 4 & 0 & 10 \end{pmatrix} = \begin{pmatrix} 404 & 190 & 324 & 160 & 162 & 514 \end{pmatrix}$$

What information would be given by multiplying the following matrices?

$$\begin{pmatrix} 404 & 190 & 324 & 160 & 162 & 514 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

- a 1) (a) At an athletics meeting, Spilman Street School gained 5 first places, 6 seconds and 3 thirds. Lowe Road School gained 4 firsts, 8 seconds and 2 thirds. Write this information as a 3×2 matrix P.
 (b) There are 5 points for a first, 3 for a second and 1 for a third. Write this as a 1×3 matrix Q.
 (c) Multiply QP to see which school won.

2) A hotel offers three classes of accommodation: A, B and C. During a certain month the number of guests and class of accommodation chosen were: Week 1, 9 class A, 6 class B, 3 class C; week 2, 10A, 2B, 5C; week 3, 6A, 4B, 6C; week 4, 6A, 5B, 7C.

- (i) Write this information as a 3×4 matrix N.
 (ii) Class A costs £14 a week, B costs £22 and C costs £27. Write this information as a 1×3 matrix M.
 (iii) Multiply MN and explain what information the answer gives.

3) (a) During a series of rugby matches the 1st XV scored 12 tries, 5 goals and 6 conversions. The 2nd XV scored 10 tries, 3 goals and 4 conversions. Write these results as a 2×3 matrix A.

- (b) A try gains 4 points, a goal 3 points and a conversion 2 points. Write this as a 3×1 matrix B.
 (c) Multiply AB and explain what the answer means.

4) (a) A tollbridge charges for each vehicle which uses it. The charge is £2 for a car, £1 for a motorcycle and £4 for a lorry or bus. Write these charges as a 1×3 matrix C.

(b) On a certain day 433 cars, 28 motorcycles and 174 lorries and buses pass over the bridge. Write these figures as a 3×1 matrix D.

- (c) Multiply CD and say what the answer means.
 (d) E is the matrix $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$. Multiply ED and say what the answer means.

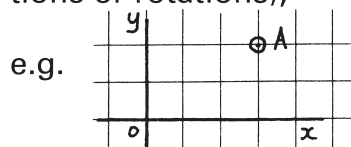
5) (a) At the sweet shop, Ben bought 3 toffee bars, 5 lollies and 12 fruit sweets; Lucy bought 4 toffee bars, 3 lollies and 7 fruit sweets. Write this information as a 2×3 matrix K.

- (b) Toffee bars are 11p each, lollies 6p each and fruit sweets 3p each. Write this as a 3×1 matrix L.
 (c) Multiply KL to give M. What information does M show?
 (d) Premultiply M by the matrix $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ to give N, and explain what information N gives.

A MATRICES (7)

Matrix transformations

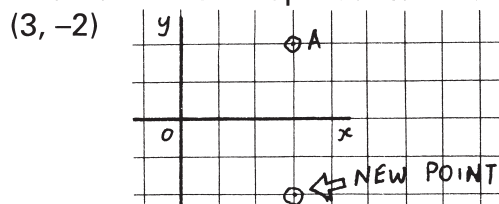
Certain matrices can be used to show transformations (reflections or rotations),



The point A in the drawing has coordinates (3, 2)

The coordinates of A may also be written as a matrix $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. By premultiplying by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ a new matrix is formed $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

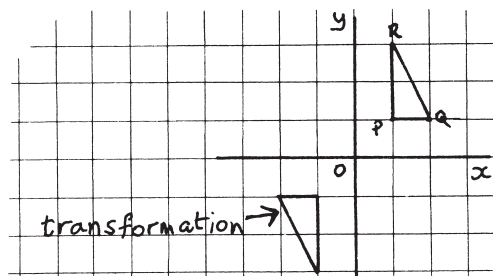
The new matrix represents a new point with coordinates



The new point is the reflection of A in the x axis.

e.g. (2) The corners of a triangle PQR have coordinates P (1, 1), Q (2, 1), R (1, 3). Write these coordinates as a 2 x 3 matrix. Then premultiply the matrix by $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

From your answer, describe how the triangle PQR has been transformed. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -1 \\ -1 & -1 & -3 \end{pmatrix}$



Triangle PQR has been rotated 180° about 0.

a Use squared paper to answer these questions (1cm squares are probably best). For each question, first draw x and y axes from -6 to +6 each.

1) Plot points A (5, 2), B (1, 4), C (1, 2). Join the points to form a triangle. Write these coordinates as a 2 x 3 matrix and premultiply by $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

Plot the new points to show the image of ABC.

Describe in words how ABC has been transformed.

2) Plot points J (3, 5), K (2, 5), L (2, 2), M (5, 2) and join to form a quadrilateral. Write the coordinates as a 2 x 4 matrix and pre multiply by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Plot the quadrilateral which is the image of JKLM and describe the transformation which JKLM has undergone.

3) Plot points D (4, 1), E (4, 4), F (6, 2). Join these points to form a triangle DEF. Write the coordinates as a matrix and pre multiply by $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Plot the coordinates given by your answer and join to form a triangle. What is the graph of the line in which DEF is reflected?

4) Repeat question 3, using the same axes, but premultiply instead by $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Describe this transformation.

5) Draw a triangle with corners (-1, 4), (-5, 2), (-3, 2) and write these coordinates as a matrix. Premultiply by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and plot the position of the new triangle given by your answer.

6) Plot points (-2, -1), (-4, -1), (-2, -2) to form a triangle A. Arrange the coordinates as a 2 x 3 matrix and premultiply by $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Draw the new triangle B.

What transformation does $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ represent?

7) Repeat question 6, using the same axes, but premultiply instead by $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. Describe this transformation.

8) Draw triangle P = $\begin{pmatrix} 2 & 4 & 2 \\ -5 & -4 & -4 \end{pmatrix}$. Premultiply by M = $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

to give Q. Draw the triangle represented by Q. Premultiply Q by N $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to give R. Draw the triangle represented by R.

By multiplying NM find the single matrix which maps P on to R. Describe the transformations P to Q, Q to R, P to R.

9) Draw triangle P (2, 1), Q (3, 1), R (3, 3) and write as a 2 x 3 matrix. Premultiply by $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and draw the result.

(a) How does the length of the new figure compare with the original?

(b) How does the area of the new figure compare with the original?

(c) What is this kind of transformation called?

10) Draw a figure with any coordinates. Write these in matrix form and premultiply by $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

What is the graph of the line produced by your answer?